Settling strongly modifies particle concentrations in wall-bounded turbulent flows even when the settling parameter is asymptotically small

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We explore the role of gravitational settling on inertial particle concentrations in a wall-bounded turbulent flow. While it may be thought that settling can be ignored when the settling parameter $\mathrm{Sv} \equiv v_s/u_\tau$ is small (v_s is Stokes settling velocity, u_τ is fluid friction velocity), we show that even in this regime the settling may make a leading order contribution to the concentration profiles. This is because the importance of settling is determined, not by the size of v_s compared with u_τ or any other fluid velocity scale, but by the size of v_s relative to the other mechanisms that control the vertical particle velocity and concentration profile. We explain this in the context of the particle mean-momentum equation, and show that in general, there always exists a region in the boundary layer where settling cannot be neglected, no matter how small Sv is (provided it is finite). Direct numerical simulations confirm the arguments and show that the near-wall concentration is highly dependent on Sv even when $\mathrm{Sv} \ll 1$, and it can reduce by an order of magnitude when Sv is increased from $O(10^{-4})$ to $O(10^{-2})$. The results also show that the preferential sampling of ejection events in the boundary layer by inertial particles when $\mathrm{Sv} = 0$ is profoundly altered as Sv is increased, and it is replaced by a preferential sampling of sweep events due to the onset of the preferential sweeping mechanism.

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I. INTRODUCTION

In many particle-laden, wall-bounded turbulent flows, it is important to characterize the near-wall distribution of a dispersed phase. In the environment, surface emission of particulate matter (e.g., dust, aerosols) is often estimated by assuming a relationship between the mean concentration and surface flux. This flux-profile relationship is also the basis for wall models of heavy scalar transport [1].

While many past studies have focused on the phenomena of turbophoresis [2,3] and the interactions of particles with near-wall coherent structures [4–6], much less attention has been given

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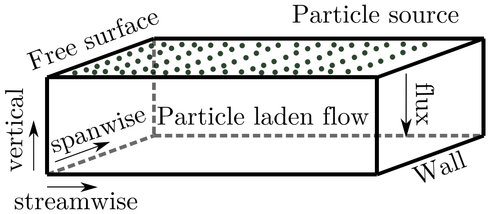

FIG. 1. Illustration of the system under consideration. The mean flow is in the horizontal direction, and gravity acts in the downward vertical direction. A source of particles at the free surface maintains a steady-state regime in which the statistics of the system depend only on the vertical position z.

to the settling of particles through wall-bounded turbulence and the impact this has on the particle concentrations. Indeed, although it is well-known that particles can experience an inertial enhancement to their settling velocity in isotropic, homogeneous turbulence [7,8], the interplay between settling and inertial effects, especially near the wall, is much less understood. In a recent study we explored this question using theory and direct numerical simulations (DNS) to consider how the mechanisms governing the particle settling and concentration profiles vary with distance from the wall [9]. It was shown that sufficiently far from the wall, the main mechanism that modifies the settling compared to the Stokes settling velocity is the enhancement due to the preferential sweeping mechanism of Maxey [7]. Closer to the wall, however, this mechanism becomes subleading and instead the turbophoretic drift velocity [2] dominates the drift of the particles toward the wall. These changes in the mechanisms controlling the particle settling velocity directly impact the particle concentration profiles, since the two quantities are connected via the particle continuity equation [9]. These results highlight the particular mechanisms that models which account for settling, but ignore particle inertia (e.g., Rouse [10], Li *et al.* [11]) must incorporate if they are to be extended to the regime of finite particle inertia.

In many studies that are aimed at understanding the behavior of inertial particles in wall-bounded turbulence, the effect of gravity is often neglected, usually under the assumption that the settling velocity is small. The relevant nondimensional parameter which describes this, however, is often assumed to be the ratio of the Stokes settling velocity $v_s \equiv \tau_p g$ (where τ_p is the particle response time and g is the gravitational acceleration) to the flow friction velocity u_τ. What we aim to demonstrate both theoretically and numerically, however, is that near the wall, even small Stokes settling velocities can be comparable or larger than the other contributions to the particle settling velocity (e.g., the turbophoretic velocity) that generate concentration buildup near the wall. This in turn disrupts commonly assumed balances which ultimately determine the wall-normal distribution of the particles. In this paper we explore this problem for the case of particles settling vertically through a horizontal turbulent boundary layer.

II. THEORETICAL ANALYSIS

We consider the vertical transport of particles in a horizontal, wall-bounded turbulent flow with a free surface. At the free surface there is a particle source, and due to gravitational settling, a downward flux of particles is established in the system. The particles are injected at the free surface at a rate that generates a steady-state regime where the statistics of the system depend only on the vertical position z. The system is illustrated in Fig. 1.

Moreover, we assume the particles are small and heavy and subject to Stokes drag and gravitational forces [12],

$$\frac{d}{dt} w^p(t) = \frac{1}{\tau_p}[u^p(t) - w^p(t)] - g, \tag{1}$$

where $w^p(t)$ is the vertical particle velocity and $u^p(t)$ is the vertical fluid velocity at the particle position. The system will also be assumed to have sufficiently low volume fraction so that one-way coupling can be assumed together with the absence of particle-particle collisions.

In assessing the importance of gravitational settling on particle motion in turbulent flows, it is typical to define a settling parameter such as $\tau_p g/u_\eta$ [13], where u_η is the Kolmogorov velocity scale] or in the context of boundary layers $\tau_p g/u_\tau$ [3], and then to conclude that the effect of settling can be neglected when these nondimensional parameters are small. Nevertheless, in some works, the effect of gravitational settling has been shown to be important even when such parameters are small. For example, DNS results in Richter and Chamecki [14], Bragg *et al.* [9] showed that even when $\tau_p g/u_\tau = O(10^{-2})$, the effect of gravitational settling on the particle motion in the boundary layer was strong, and in some parameter regimes, made a leading order contribution to the particle motion. As we will now show, this is because quantities such as $\tau_p g/u_\tau$ are inappropriate measures of the importance of settling on the particle motion in the boundary layer.

In the following, all quantities are normalized using the fluid friction timescale τ and velocity scale u_τ to express them in wall units, usually denoted by the superscript "+". However, in what follows, we drop the superscript for notational simplicity.

Using phase-space probability density function (PDF) equations, we can construct transport equations for the average concentration $\varrho \equiv \langle \delta(z^p(t) - z) \rangle$, where $z^p(t)$ is the vertical particle position, z is the time-independent vertical position coordinate (with $z = 0$ corresponding to the wall), and $\langle \cdot \rangle_z$ denotes an ensemble average conditioned on $z^p(t) = z$. The continuity equation governing $\varrho(z, t)$ is [9]

$$\partial_t \varrho + \nabla_z [\varrho \langle w^p(t) \rangle_z] = 0. \tag{2}$$

Equation (2) can be solved by specifying $\langle w^p(t) \rangle_z$, for which an expression can be obtained from the particle mean-momentum equation [9]

$$\langle w^p(t) \rangle_z = \langle u^p(t) \rangle_z - \mathrm{Sv} - \mathrm{St}(D_t \langle w^p(t) \rangle_z + \nabla_z \mathcal{W} + \mathcal{W} \varrho^{-1} \nabla_z \varrho), \tag{3}$$

where $D_t \equiv \partial_t + \langle w^p(t) \rangle_z \nabla_z$, $\mathcal{W} \equiv \langle (w^p(t) - \langle w^p(t) \rangle_z)^2 \rangle_z$ is the variance of the vertical particle velocity, $\mathrm{St} \equiv \tau_p/\tau$ is the Stokes number, $\mathrm{Sv} \equiv \tau_p g/u_\tau$ is the settling number. Hereafter, the mean vertical velocity $\langle w^p(t) \rangle_z$ will sometimes be referred to as the settling velocity, which is to be distinguished from the Stokes settling velocity Sv.

The term $\langle u^p(t) \rangle_z$ in Eq. (3) is a mean velocity that arises from the particles preferentially sampling the underlying turbulent flow [3], that vanishes for fully mixed fluid particles [15]. The term $D_t \langle w^p(t) \rangle_z$ is the mean particle acceleration. The term $-\mathrm{St} \nabla_z \mathcal{W}$ is the turbophoretic velocity which arises due to the combination of turbulence inhomogeneity and particle inertia [2]. Finally, the term $-\mathrm{St} \mathcal{W} \varrho^{-1} \nabla_z \varrho$ is a diffusive velocity that arises from decoupling between the particle velocity and the local fluid velocity, and is only finite when the concentration field is nonuniform. Detailed explanations of each of the terms in Eq. (3) may be found in Bragg *et al.* [9].

For the steady-state regime of interest the solution to Eq. (2) is that the flux $\varrho \langle w^p(t) \rangle_z$ is a constant (that depends upon the boundary conditions and system parameters). For $\mathrm{Sv} = 0$, a possible steady-state is the zero-flux configuration, for which $\langle w^p(t) \rangle_z = 0$. For $\mathrm{Sv} > 0$, unless resuspension mechanisms at the wall are sufficiently strong to overcome the weight of the particles, a zero-flux configuration will not be established, and instead a constant, negative flux will be established with $\langle w^p(t) \rangle_z < 0$. In this paper we consider this constant negative flux regime, although the analysis and its implications could easily be extended to the zero-flux configuration.

The regime $\mathrm{St} \ll 1$ but finite Sv corresponds to the regime of negligible particle inertia, but nonnegligible settling that was analyzed by Rouse [10]. Our interest, by contrast, is the regime $\mathrm{Sv} \ll 1$ and finite St, and to understand whether in this regime the contribution from Sv to the total vertical velocity $\langle w^p(t) \rangle_z$ in Eq. (3) may be ignored. When the contribution of Sv to $\langle w^p(t) \rangle_z$ can be ignored, then it follows from Eq. (2) that the concentration profile will be independent of Sv. Equation (3) is regularly perturbed with respect to Sv, i.e., the limit of (3) for $\mathrm{Sv} \to 0$ is equal to Eq. (3) when setting $\mathrm{Sv}=0$. To explore if Sv may be neglected if it is small but nonzero we introduce

$$\Lambda(z) \equiv \langle u^p(t)\rangle_z - \mathrm{St}(D_t\langle w^p(t)\rangle_z + \nabla_z\mathcal{W} + \mathcal{W}\varrho^{-1}\nabla_z\varrho), \tag{4}$$

so that $\langle w^p(t)\rangle_z = \Lambda - \mathrm{Sv}$, and expand Λ in Sv to obtain

$$\Lambda(z) = \sum_{n=0}^{\infty}\mathrm{Sv}^n\Lambda_n(z), \tag{5}$$

$$\langle w^p(t)\rangle_z = \Lambda_0(z) + \mathrm{Sv}(\Lambda_1(z)-1) + O(\mathrm{Sv}^2), \tag{6}$$

where $\Lambda_0(z) = \Lambda(z)|_{\mathrm{Sv}=0}$. For settling to be ignored we require that $\Lambda_0(z) \gg O(\mathrm{Sv})$. To consider when this condition is satisfied, we will first focus on the regime $z \ll 1$ where analytical results are possible, and which is also the regime of most interest since this is where the particle concentration is highest. It is also worth emphasizing that although the following analysis is strictly for the limited regime $z \ll 1$, the asymptotic results to be used in the analysis are known to hold up to $z = O(10)$ [3,16], and therefore our results should also apply up to $z = O(10)$.

To show whether the condition $\Lambda_0(z) \gg O(\mathrm{Sv})$ is satisfied, we must describe each of the terms contributing to $\Lambda_0(z)$ in the near wall region. It was shown in Sikovsky [16] using asymptotic analysis that for the case of finite particle mass-flux with $z \ll 1$ and $\mathrm{Sv}=0$, $\varrho \sim z^{-\gamma}$ (in what follows we ignore the coefficients in the asymptotic relationships since it is the dependence on z that will be of interest), where $\gamma = \max[\alpha_0, 3]$ and $\alpha_0(\mathrm{St}) \in [0,4]$, so that $\gamma(\mathrm{St}) \in [3,4]$ [17]. The asymptotic analysis also shows that for sufficiently large St, an additional contribution in the expansion for ϱ becomes important which generates $\varrho \sim$ constant in the limit $\mathrm{St} \to \infty$, corresponding to particles moving ballistically through the flow. For simplicity, we will restrict our focus in this analysis to low to moderate inertia particles for which $\varrho \sim z^{-\gamma}$ describes the correct behavior. Since $\varrho\langle w^p(t)\rangle_z$ is constant, the result $\varrho \sim z^{-\gamma}$ then implies $\langle w^p(t)\rangle_z \sim z^{\gamma}$ and $D_t\langle w^p(t)\rangle_z \sim z^{2\gamma-1}$. Using these results, we will now consider whether the condition $\Lambda_0(z) \gg O(\mathrm{Sv})$ is satisfied for weak to moderate inertia particles.

In the weak inertia regime $\mathrm{St} \ll 1$ with $\mathrm{Sv}=0$, $\mathcal{W} \sim z^4$ [3]. Furthermore, the model in Sikovsky [16] for $\langle u^p(t)\rangle_z$ yields the behavior $\langle u^p(t)\rangle_z \sim z^3$ for arbitrary St in the regime where $\varrho \sim z^{-\gamma}$. Therefore, substituting these asymptotic results into the definition of Λ_0 we obtain

$$\Lambda_0(z) \sim z^3 - \mathrm{St}(z^{2\gamma-1} + z^3 + z^3). \tag{7}$$

For $\mathrm{St} \ll 1$, $\gamma = 3$ [16], and so Eq. (7) has the limiting form $\Lambda_0 \sim z^3$. This shows that the condition $\Lambda_0(z) \gg O(\mathrm{Sv})$, which must be satisfied if settling is to be ignored, will be violated for $z \to 0$ if $\mathrm{Sv} > 0$. Hence, for $\mathrm{St} \ll 1$ there will always exist a region near the wall where gravitational settling cannot be ignored, even if $\mathrm{Sv} \ll 1$ (but finite).

For particles with moderate inertia and for $z \ll 1$, Sikovsky [16] showed that the particle velocity moments obey the asymptotic result $\langle (w^p(t) - \langle w^p(t)\rangle_z)^n\rangle_z \sim z^{\gamma}$ for $n \geqslant 2$, and therefore $\mathcal{W} \sim z^{\gamma}$. Using this we obtain

$$\Lambda_0(z) \sim z^3 - \mathrm{St}(z^{2\gamma-1} + z^{\gamma-1} + z^{\gamma-1}). \tag{8}$$

Since $\gamma \in [3,4]$, the leading behavior is $\Lambda_0 \sim z^{\gamma-1}$. Therefore, just as for the $\mathrm{St} \ll 1$ case, this also shows that the condition $\Lambda_0(z) \gg O(\mathrm{Sv})$ will be violated for moderately inertial particles for $z \to 0$ if $\mathrm{Sv} > 0$.

We have therefore shown that for both weakly and moderately inertial particles, even if $\mathrm{Sv} \ll 1$, there will always exist a region close to the wall where settling cannot be ignored, and indeed where settling makes a leading order contribution to $\langle w^p(t)\rangle_z$ and therefore ϱ. Further away from the wall it is also possible that the condition $\Lambda_0(z) \gg O(\mathrm{Sv})$ could be violated since $\Lambda_0(z)$ depends on gradients in the flow statistics, and these become weak for sufficiently large z (at least if the flow Reynolds number is large enough for a quasi-homogeneous region to emerge sufficiently far from the wall). To explore the role of settling on the particle dynamics throughout the boundary layer, we will now consider DNS results for inertial particle motion in an open channel flow. These results will enable us to go beyond the analysis in this section by allowing for a quantitative assessment of the impact of settling on the particle concentration profiles in the regime $\mathrm{Sv} \ll 1$.

III. DIRECT NUMERICAL SIMULATIONS

DNS is used to solve the incompressible Navier-Stokes equations, which are then used in a one-way coupled scenario to solve for the motion of heavy, inertial, point-particles that are subject to drag and gravitational forces via the equation of motion

$$\frac{d^2}{dt^2}\boldsymbol{x}^p(t) \equiv \frac{d}{dt}\boldsymbol{v}^p(t) = \frac{1}{\tau_p}\Big[\boldsymbol{u}^p(t) - \boldsymbol{v}^p(t)\Big] - \boldsymbol{g}, \tag{9}$$

where $\boldsymbol{x}^p(t)$, $\boldsymbol{v}^p(t)$ are the particle position and velocity, $\boldsymbol{u}^p(t)$ is the fluid velocity at the particle position, and $\boldsymbol{g}$ is the gravitational acceleration in the vertical direction. The flow has a friction Reynolds number of $\mathrm{Re}_\tau = 315$ and is generated by applying a constant streamwise pressure gradient to force the flow. The streamwise x and spanwise y directions are periodic, and the wall at $z = 0$ imposes a no-slip condition on the fluid velocity field. At the upper wall, $z = 315$, a free-slip (i.e., zero-stress) condition is imposed on the fluid velocity. The domain size is $L_x \times L_y \times L_z = 1979 \times 1979 \times 315$ with a corresponding grid of $N_x \times N_y \times N_z = 128 \times 256 \times 128$. The grid is stretched in the wall-normal direction and thus the simulations have a resolution of $\Delta_x \times \Delta_y \times \Delta_z = 15.46 \times 7.73 \times 0.25\,(\text{wall}), 4.49\,(\text{center})$. The unladen flow field from DNS has been tested and validated by comparison with published data for $\mathrm{Re}_\tau = 40$–950 [18,19]. Inertial particles are introduced into the flow at $z = 315$ with an initial velocity equal to the Stokes settling velocity plus the local fluid velocity. When the particles eventually settle down to the wall (i.e., when the distance between their centroid and the wall is equal to the particle radius) they are removed and replaced by another particle injected at $z = 315$. This system is then simulated until the flow reaches steady state. To measure the particle concentration, the computational domain is divided into 256 slabs in the wall-normal direction. A particle is considered to belong to a particular slab when its centroid is located within that slab. The point-particle DNS code has been validated for inertial particles in the range $\mathrm{St} = 30$–2000 [20]. This setup provides a canonical case of steady-state, particle-laden wall-bounded turbulence which corresponds to the system described by the theory that was illustrated in Fig. 1.

To test the idea presented in the previous section, it would be desirable to compare results for $\mathrm{Sv} = 0$ with those from $0 < \mathrm{Sv} \ll 1$ to see the impact of settling when Sv is small. However, at finite St and with the absorbing wall boundary condition described above, particles accumulate in the vicinity of the wall and free-surface for simulations using $\mathrm{Sv} = 0$, since there exists no gravitational settling with which to exit the viscous layer, and resuspension events are rare. This results in a situation where the simulations cannot reach a steady state in a reasonable amount of time. Therefore, we instead consider results for finite Sv in the range $0 < \mathrm{Sv} \ll 1$. If the results are insensitive to Sv then this would show that settling can be ignored in this regime. If, however, there are regions of the flow where the results are sensitive to Sv even when $\mathrm{Sv} \ll 1$, then this would confirm the prediction of the theoretical analysis in the previous section. In view of this, we consider simulations with $\mathrm{Sv} = 3 \times 10^{-4},\ 3 \times 10^{-3},\ 3 \times 10^{-2},\ 3 \times 10^{-1}$, and for each of these we consider four different Stokes numbers $\mathrm{St} = 0.93,\ 2.80,\ 9.32,\ 46.67$. In the simulations the same particle diameter ($d_p = 0.236$) was used for each St, such that effectively the particle density was used to vary St. Roughly speaking, the chosen values of Sv would correspond to a sand/dust grain with a diameter of $O(1$–$100\ \mu\mathrm{m})$ suspended over a windy surface (e.g., $u_\tau \approx O(0.1\ \mathrm{m/s})$. Note that $\mathrm{Sv} = 3 \times 10^{-1}$ could be considered to be too large to satisfy the requirement $\mathrm{Sv} \ll 1$; we nevertheless include it to better identify the limits of the above theory.

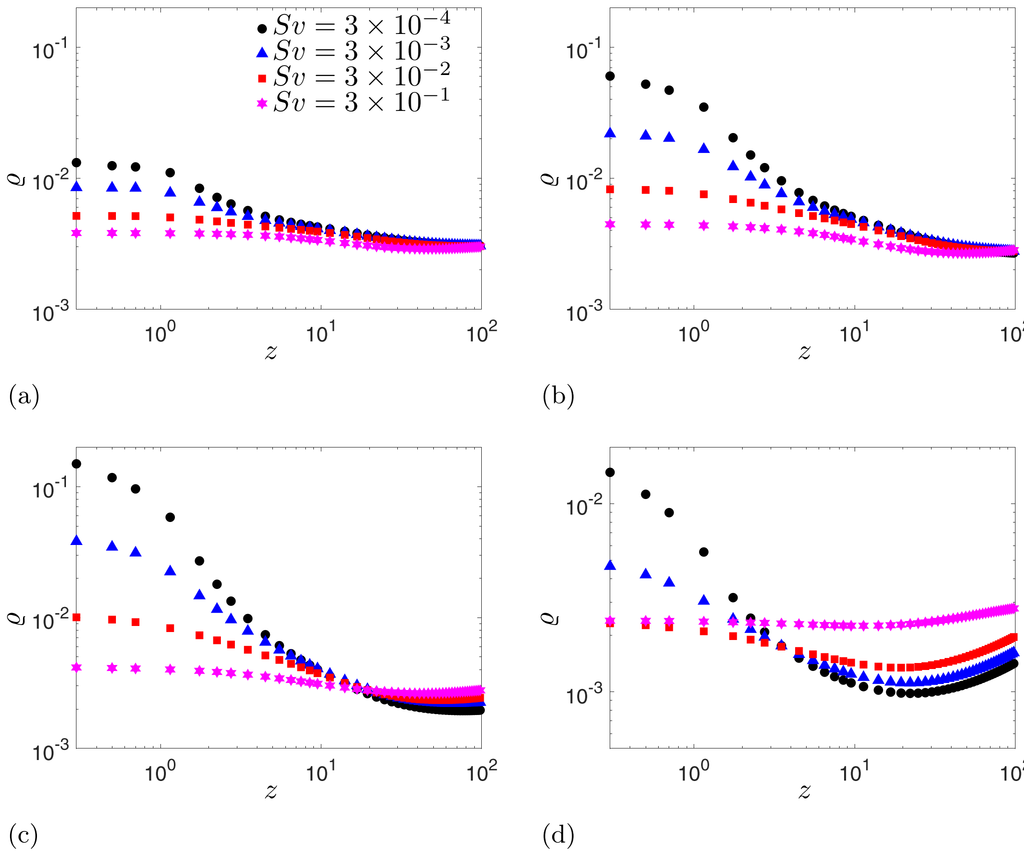

FIG. 2. Plot of ϱ as a function of z for different Sv, and (a) St = 0.93, (b) St = 2.80, (c) St = 9.30, (d) St = 46.5.

IV. RESULTS AND DISCUSSION

We begin by considering in Fig. 2 the results for concentration $\varrho(z)$ for different Sv and St (we remind the reader that all quantities are in wall-units, with the usual "+" superscript dropped for notational simplicity). For each St, the results show a very strong effect of Sv on ϱ, with strong reductions in the near-wall concentration as Sv is increased. Indeed, for the cases with St = 2.8, 9.3, 46.5, the near wall value of ϱ reduces by an order of magnitude as Sv is increased from $\mathrm{Sv} = 3 \times 10^{-4}$ to $\mathrm{Sv} = 3 \times 10^{-2}$. This remarkable sensitivity of ϱ to Sv even when $\mathrm{Sv} \ll 1$ confirms the prediction of the analysis in Sec. II. Conceptually, that gravity leads to a reduction in the value of the concentration near the wall is because with an absorbing wall, settling reduces the residence time of the particles in the near-wall region as their vertical velocity through the viscous sublayer is larger than it would be in the absence of settling. For the case of particles which elastically collide with the wall, the steady-state would be characterized by zero flux ($\langle w^p(t)\rangle_z = 0$), and settling would actually lead to an increase of the near wall concentration in general since their weight would keep them trapped in the near wall region until they experience large enough fluctuations of the vertical fluid velocity to resuspend them into the flow. The analysis of Sec. II can also be extended to this zero-flux case and would again show that even if $\mathrm{Sv} \ll 1$, settling can make a nonnegligible contribution to the near wall concentration of the particles (this has been confirmed in results not shown here).

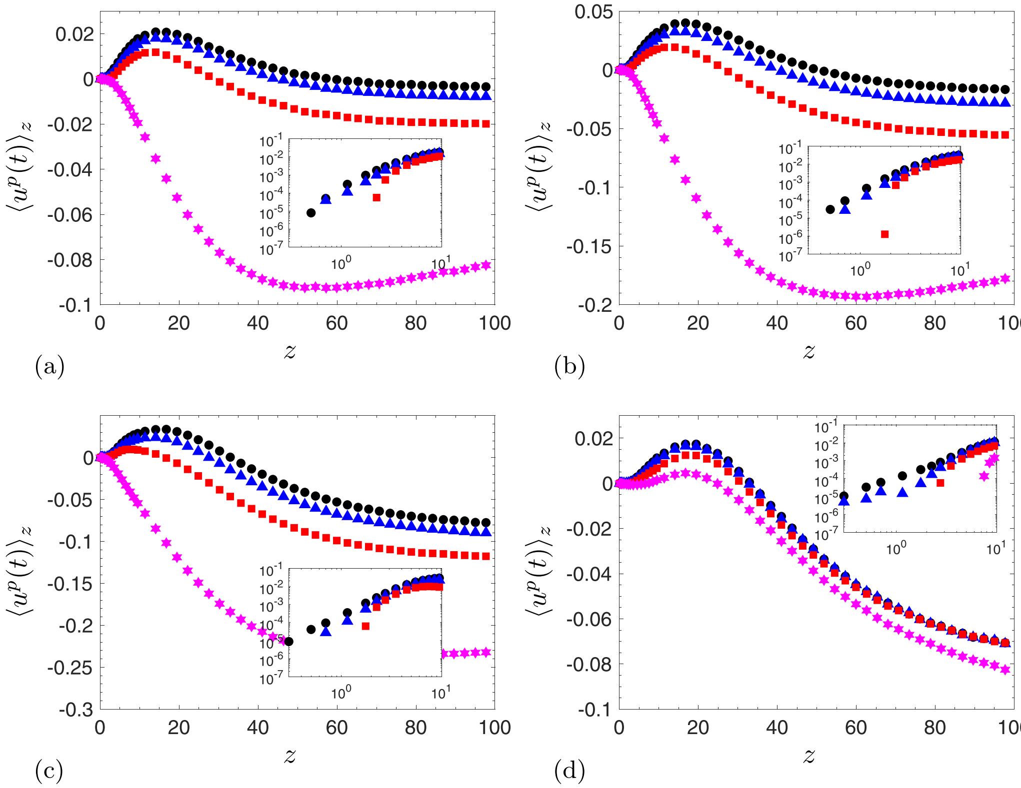

FIG. 3. Plots of the average fluid velocity sampled by the particles $\langle u^p(t)\rangle_z$ as a function of z for different Sv, and (a) St = 0.93, (b) St = 2.80, (c) St = 9.30, (d) St = 46.5. The insets correspond to the same plots but in a log-log scale to highlight the behavior for small z (and in these plots the symbols disappear for small z once the quantity becomes negative). Legend is the same as that in Fig. 2.

Not only do the results in Fig. 2 show a strong effect of settling on the magnitude of the particle concentrations even when Sv $\ll 1$, but they also indicate that settling affects the scaling behavior of $\varrho(z)$. In particular, as discussed in Sec. II, the asymptotic analysis of Sikovsky [16] shows that for Sv $= 0$, as the wall is approached the concentration behaves as a power law $\varrho(z) \sim z^{-\gamma}$, which was also confirmed using DNS data. Unlike those DNS results (or those in Johnson *et al.* [3]), the results in Fig. 2 show that as z decreases, there is a noticeable kink in the curves for ϱ around $z = O(1)$, below which ϱ appears to flatten out. This change in behavior is due to the role of Sv in Eq. (3).

We now turn to consider how settling affects each of the terms in $\Lambda(z)$ as defined in Eq. (4). In Fig. 3, the results for $\langle u^p(t)\rangle_z$ are shown for each of the St, Sv combinations. In the absence of settling, this term is known to be positive in the near wall region [3], which is associated with the preference of the particles to accumulate in the near-wall ejection events of the turbulent boundary layer [4,5,21]. Our results also show $\langle u^p(t)\rangle_z > 0$ near the wall for the smallest Sv case Sv $= 3 \times 10^{-4}$. However, as Sv is increased, the size of $\langle u^p(t)\rangle_z$ reduces, and is negative near the wall for Sv $= 3 \times 10^{-1}$. Moreover, the inset plots in Fig. 3 indicate that even for Sv $= 3 \times 10^{-2}$, $\langle u^p(t)\rangle_z$ is negative for $z \leqslant O(1)$ (at which point the curve disappears on the logarithmic plot). Hence, the preferential sampling of ejection events in the boundary layer by the inertial particles is very sensitive to Sv even when Sv $\ll 1$, and transitions to a preferential sampling of sweep events as Sv increases (over a range of z that depends on St and Sv). This striking change in behavior can be understood as being due to the onset of the preferential sweeping mechanism [7,22] as Sv increases, and has been seen in other DNS of particles settling through wall-bounded turbulence [23]. This

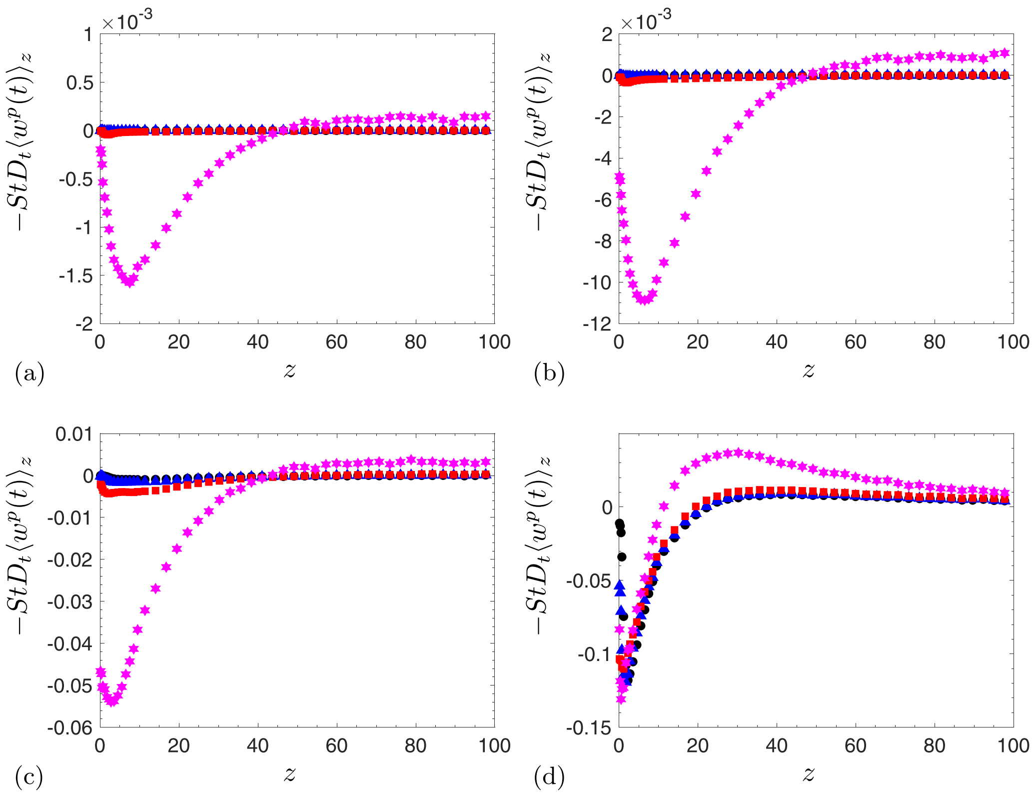

FIG. 4. Plots of the average vertical particle velocity $\langle w^p(t)\rangle_z$ as a function of z for different Sv, and (a) St = 0.93, (b) St = 2.80, (c) St = 9.30, (d) St = 46.5. Legend is the same as that in Fig. 2. Note that inset plots showing the results in a log-log scale are not shown for this quantity as the data is very noisy for small z due to the spatial differentiation of the data for $\langle w^p(t)\rangle_z$ involved in computing $D_t\langle w^p(t)\rangle_z$.

change in the sign of $\langle u^p(t)\rangle_z$ as Sv increases also means that the role of this term in governing the particle concentration changes. When $\langle u^p(t)\rangle_z > 0$ this term hinders the settling of the particles toward the wall and acts to reduce their near wall accumulation [3]. However, when $\langle u^p(t)\rangle_z < 0$, this term contributes to the settling of the particles toward the wall and so can contribute to their near wall accumulation.

In Fig. 4, the results for $-\mathrm{St}D_t\langle w^p(t)\rangle_z$ are shown for each of the St, Sv combinations. Since the flow has constant negative flux $\varrho\langle w^p(t)\rangle_z < 0$, then since ϱ increases as the wall is approached, it follows that $-\mathrm{St}D_t\langle w^p(t)\rangle_z$ is negative, as shown in the results. In Bragg *et al.* [9] it was shown that for $\mathrm{Sv} = 3 \times 10^{-2}$ this acceleration term makes a negligible contribution to $\langle w^p(t)\rangle_z$. Our results for different Sv reveal that although this term generally makes a small contribution to $\langle w^p(t)\rangle_z$, it can become important for large St as Sv increases. However, for these larger St, Sv cases, the concentration is almost uniform (see Fig. 2), and so while $-\mathrm{St}D_t\langle w^p(t)\rangle_z$ can be important for predicting the settling velocity $\langle w^p(t)\rangle_z$, it is not important in the regime of St, Sv for which there is a significant buildup of particle concentration near the wall.

In Fig. 5, the results for the turbophoretic drift velocity $-\mathrm{St}\nabla_z\mathcal{W}$ are shown for different St and Sv. Compared to the results for $\langle u^p(t)\rangle_z$ and $-\mathrm{St}D_t\langle w^p(t)\rangle_z$, the results show that $-\mathrm{St}\nabla_z\mathcal{W}$ is relatively insensitive to Sv. The insets to the plots reveal, however, that there is a strong dependency on Sv in the near wall region $z \leqslant O(1)$, with $-\mathrm{St}\nabla_z\mathcal{W}$ increasing as Sv increases for $\mathrm{Sv} < 3 \times 10^{-1}$. The corresponding results for $\mathcal{W}$ (not shown) also show that $\mathcal{W}$ increases as Sv increases. This behavior may be partially understood by considering the expression for $\mathcal{W}$ obtained using the

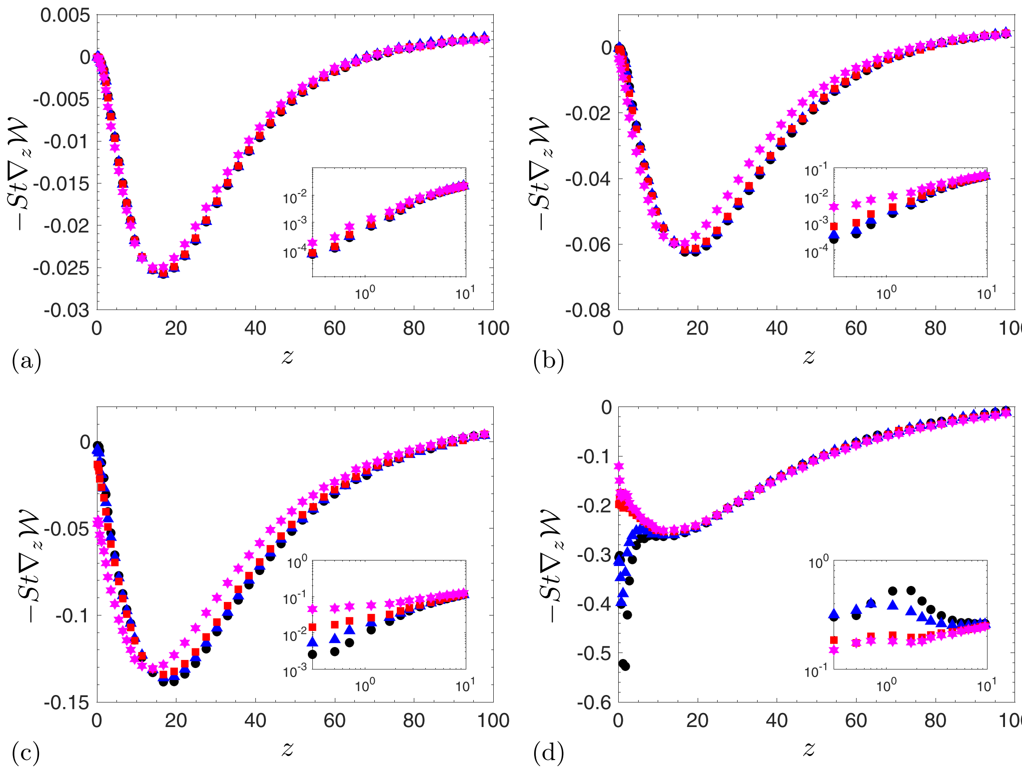

FIG. 5. Plots of the turbophoretic velocity $-\mathrm{St}\nabla_z\mathcal{W}$ as a function of z for different Sv, and (a) St = 0.93, (b) St = 2.80, (c) St = 9.30, (d) St = 46.5. The insets correspond to the same plots (except that the vertical axis is now $\mathrm{St}\nabla_z\mathcal{W}$) but in a log-log scale to highlight the behavior for small z. Legend is the same as that in Fig. 2.

formal solution to Eq. (9), namely,

$$\mathcal{W}(z) = \int_{-\infty}^{0}\int_{-\infty}^{0} e^{(s+s')/\mathrm{St}}(\langle u^p(s)u^p(s')\rangle_z - \langle u^p(s)\rangle_z\langle u^p(s')\rangle_z)\,ds'\,ds. \tag{10}$$

Settling reduces the correlation timescale of the flow seen by the particle [24], meaning that increasing Sv will reduce the time span $|s-s'|$ over which the covariance $\langle u^p(s)u^p(s')\rangle_z$ is finite. This in turn would have the effect of reducing $\mathcal{W}$ with increasing Sv (see Ref. [25] for the discussion of an analogous effect in the context of how settling impacts particle-pair relative velocities). The results in Fig. 3 show, however, that increasing Sv leads to a reduction in the preferential sampling of the flow for $\mathrm{Sv} \leqslant 3\times10^{-2}$. If this reduction in the preferential sampling leads to a greater reduction in $\langle u^p(s)\rangle_z\langle u^p(s')\rangle_z$ than the reduction in the correlation timescale associated with $\langle u^p(s)u^p(s')\rangle_z$, then according to Eq. (10) the overall effect of increasing Sv would be to increase $\mathcal{W}$, as observed. Such an argument also shows the way in which the preferential sampling of the flow impacts $\langle w^p(t)\rangle_z$ not only explicitly through the term $\langle u^p(t)\rangle_z$ in Eq. (3) but also implicitly through its effect on $-\mathrm{St}\nabla_z\mathcal{W}$, which itself depends on how the particles interact with and sample the flow.

Finally, in Fig. 6, the results for the diffusive velocity $-\mathrm{St}\mathcal{W}\varrho^{-1}\nabla_z\varrho$ are shown for different St and Sv. Near the wall this velocity is positive since $\nabla_z\varrho < 0$ there, so that this diffusive velocity hinders the settling velocity of the particles. This term is also very sensitive to Sv even for $\mathrm{Sv} \ll 1$, which is mainly due to the sensitivity of ϱ to Sv that was shown in Fig. 2. Throughout much of the boundary layer this term makes a subleading contribution in Eq. (3). However, comparing the

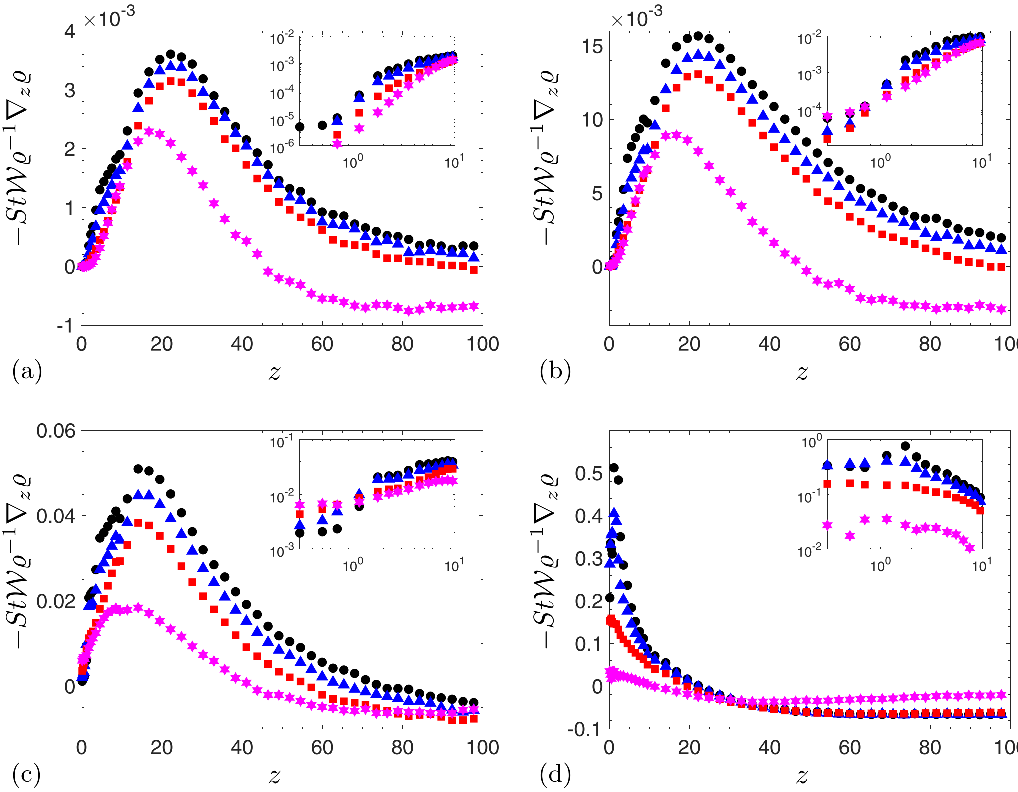

FIG. 6. Plots of the diffusive velocity $-\mathrm{St}\mathcal{W}\varrho^{-1}\nabla_z\varrho$ as a function of z for different Sv, and (a) St = 0.93, (b) St = 2.80, (c) St = 9.30, (d) St = 46.5. The insets correspond to the same plots but in a log-log scale to highlight the behavior for small z. Legend is the same as that in Fig. 2.

logarithmic scale inset plots to those of the other quantities reveals that very close to the wall where the gradients of ϱ are strongest, this term can be significant compared to the other contributions in Eq. (3).

V. CONCLUSIONS

We have considered the role of gravitational settling on the concentration profiles of small inertial particles in wall-bounded turbulent flows. We provided theoretical arguments and DNS results that show that settling can play a leading order role in determining the concentrations, even when the Stokes settling velocity is very small compared with the fluid friction velocity. The reason is that the dynamical relevance of settling is determined by the size of the Stokes settling velocity compared with the other mechanisms contributing to the particle vertical velocity, not compared with the fluid friction velocity (or any other fluid velocity scale). In the theoretical analysis, this corresponds to saying that settling can only be ignored if $\Lambda_0 \equiv \Lambda|_{\mathrm{Sv}=0}$ [see Eq. (4) for the definition of $\Lambda(z)$] is much larger than $O(\mathrm{Sv})$. However, as we have shown, this condition will always be violated for $\mathrm{Sv} > 0$ in some region close to the wall since in the viscous sublayer, $\lim_{z\to 0}\Lambda_0 \to 0$.

Quantitatively, the DNS results showed that the near-wall concentration is highly dependent on Sv even when $\mathrm{Sv} \ll 1$, and indeed the concentration can be reduced by an order of magnitude when Sv is increased from $O(10^{-4})$ to $O(10^{-2})$. The results also show that the preferential sampling of ejection events in the boundary layer by inertial particles that occurs for $\mathrm{Sv} = 0$ is profoundly altered as Sv is increased, and is replaced by a preferential sampling of sweeping events due to the onset of the preferential sweeping mechanism. The results are very consequential for understanding and predicting the concentration profiles of inertial particles in wall-bounded turbulent flows, since many previous studies neglected the effect of settling under the assumption that its effect is negligible for $\mathrm{Sv} \ll 1$.

One practical issue is that it is not always possible to reliably predict $\Lambda_0(z)$ *a priori*, and therefore when performing numerical simulations, it may not be clear whether the condition $\Lambda_0 \gg O(\mathrm{Sv})$ will be satisfied so as to justify neglecting particle settling. Our results suggest that it is probably best to always retain the particle settling, and then one can check *a posteriori* whether the settling plays any important role.

The analysis in this paper made several assumptions that deserve further investigation. First, in environmental flows the surface typically has a finite roughness which was ignored in our analysis and DNS. Given that the main impact of settling discussed in this paper is in the viscous sublayer of the flow (though its effect is not confined to this region for larger St particles), the finite roughness of real environmental surfaces could have implications for the importance of settling in these contexts. However, we remind the reader that the viscous sublayer asymptotics employed in the theory is quite accurate up to $z = O(10)$, even though it is only formally valid for $z \ll 1$, as discussed in Ref. [9]. As a result, even with a rough surface, there can still be a significant region close to the surface where the theoretical analysis applies and where the effect of settling can still be crucial even when $\mathrm{Sv} \ll 1$.

Second is our assumption that gravity acts perpendicular to the surface in the wall-bounded flow, whereas in engineered contexts gravity is often assumed to act parallel to the surface in the direction of the mean flow (e.g., Ref. [26]). If the angle between the direction of gravity and the surface normal is given by θ, then it is straightforward to show that as θ is increased from $\theta = 0$ (the case we have considered) toward the limit $|\theta| = \pi/2$, then there continues to be a finite region close to the wall where settling cannot be ignored, although the extent of this region reduces as $|\theta|$ is increased. The effect only disappears when $|\theta| = \pi/2$, corresponding to gravity acting in the streamwise direction. For the limit $|\theta| = \pi/2$ the specific effect we have discussed disappears since in this limit the wall-normal Stokes settling velocity in Eq. (3) would be identically zero. However, it does not follow from this that settling can therefore be ignored because the implicit of gravity might still be very important. In particular, recall that each of the terms on the right-hand side of Eq. (3) are affected by settling—the Sv term explicitly and the other terms implicitly, due to the way settling modifies the particle interaction with the flow field. If $|\theta| = \pi/2$, then sufficiently close to the wall the streamwise fluid velocity will be much smaller than the settling velocity in that direction, and hence near the wall the particle trajectory in the streamwise direction will be strongly affected by the settling. This could then strongly modify the way the particles interact with the near-wall flow, and hence settling could strongly affect all of the terms on the right-hand side of Eq. (3) that depend implicitly upon the settling. This is an interesting point that should be explored in future work.

Third is our assumption that the particles are absorbed at the boundary. For this case, we showed that settling can strongly suppress the near-wall particle concentration even when $\mathrm{Sv} \ll 1$. Our analysis could also be extended to the case where particles rebound at the wall. For this situation, settling would lead to an enhancement of the near-wall particle concentration even when $\mathrm{Sv} \ll 1$ since the effect of settling would be to trap the particles in the near-wall region for extended periods of time before they are resuspended by the flow.

Finally, it is interesting to note that in Ref. [27] the effect of electric forces on the dynamics of inertial particles in a turbulent channel flow was considered. They found that even when the nondimensional number characterizing the electric charge is small, the effect of the charging on the particle dynamics can be large in the near-wall region. This effect is analagous to the strong effect of settling even when $\mathrm{Sv} \ll 1$ that we have considered in this paper, and a similar analysis to that which we have presented could be applied to the system considered in Ref. [27] to explain those results.

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